AP® Calculus BC
2002 Sample Student Responses
Form B

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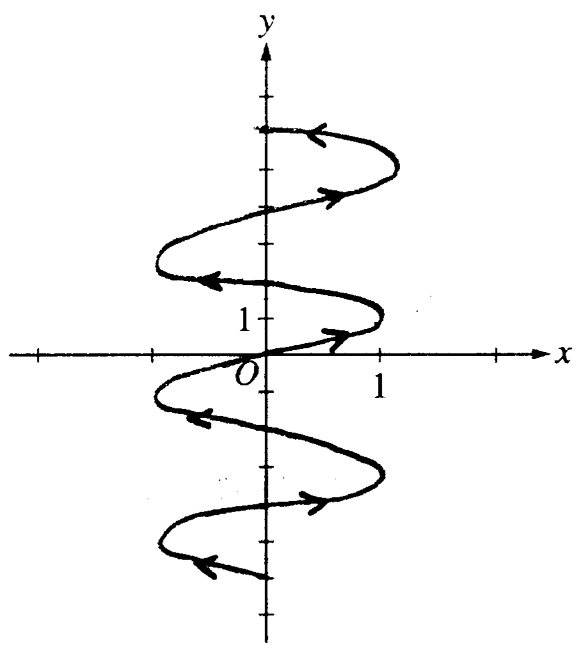
A1

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)



Work for problem 1(b)

Range of $x(t)$: $-1 \leq x \leq 1$

Range of $y(t)$: $-2\pi \leq y \leq 2\pi$

A2

Work for problem 1(c)

$x(t) = \sin(3t)$

$\frac{dx}{dt} = 3\cos(3t)$

Cr. point: $3\cos(3t) = 0$

$\cos(3t) = 0.$

$3t = \pi/2$

$t = \pi/6$

$\frac{d^2x}{dt^2} = -9\sin(3t)$

$\frac{d^2x}{dt^2}\Big|_{t=\pi/6} = -9\sin(\pi/2) < 0 \Rightarrow$ $t = \pi/6$ is at a $\therefore$ Local maximum

Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$\frac{dx}{dt}\Big|_{t=\pi/6} = 3\cos(\pi/2) = 0.$

$\frac{dy}{dt} = 2$ for all t.

Speed at $\pi/6 = \sqrt{0 + 2^2} = 2$ units/

Work for problem 1(d)

Distance travelled $= \int_{-\pi}^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_{-\pi}^{\pi} \sqrt{9\cos^2(3t) + 4}\, dt \approx 17.973$

$5\pi \approx 15.708$

$\therefore$ Distance traveled from $t = -\pi$ to $t = \pi$ is greater than 5π.

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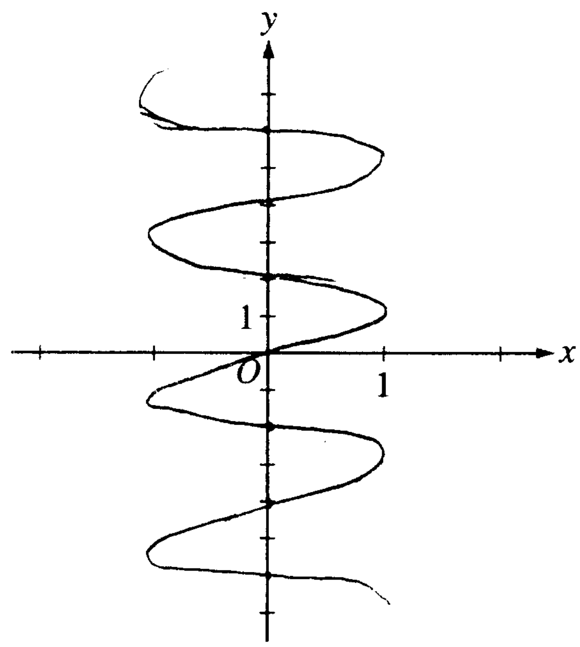
C1

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)



Work for problem 1(b)

$x(t) = \sin(3t)$ $= -1 \le \sin \le 1$

$-1 \le x \le 1$

$y(t) = 2t$

y—all real numbers

C2

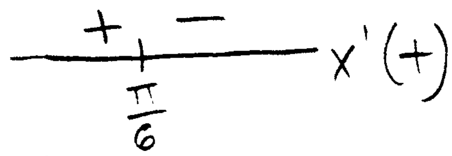
Work for problem 1(c)

$x'(t) = 3\cos(3t)$

$3\cos(3t) = 0$

$3t = \frac{\pi}{2}$ or multiple

$\boxed{t = \frac{\pi}{6}}$ – smallest



speed $= +\sqrt{(y')^2 + (x')^2} = \sqrt{2^2 + 9\cos^2(3t)}$

$= \sqrt{4 + 0}$

$= \boxed{2 \text{ un/time unit}}$

Work for problem 1(d)

$\text{dist} = \int_{-\pi}^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_{-\pi}^{\pi} \sqrt{9\cos^2(3t) + 4}\, dt$

$= 17.973$ un

$5\pi = 15.708 < 17.973$

yes

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A,

Work for problem 2(a)

a) $P'(9) = 1 - 3(e)^{-0.2\sqrt{9}}$

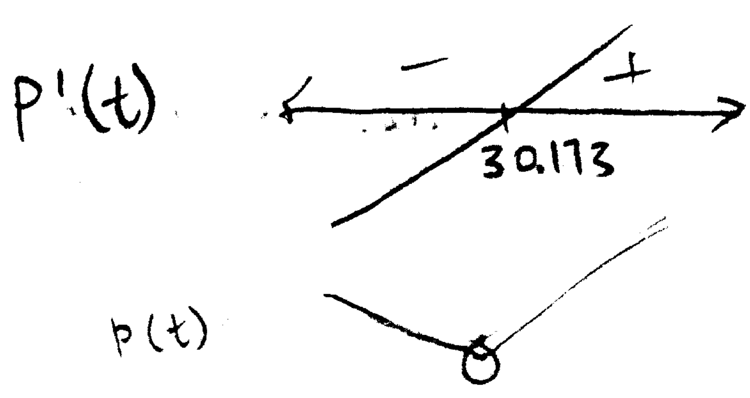
$= -0.646$ gallons/day.

No. $P'(9)$ is negative, so the amount of pollutant is decreasing.

Work for problem 2(b)

b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$

$t = 30.173$



minimum at $t = 30.173$

A₂

Work for problem 2(c)

$$50 + \int_0^{30.173} p'(t)\, dt$$

$$= 50 - 14.895$$

$$= 35.104 \text{ gallons}$$

At day 30, there will be 35.104 gallons of pollutant left, and 35.104 < 40, ∴ it will be safe.

Work for problem 2(d)

$$p'(0) = 1 - 3e^{-0.2\,0}$$

$$= 1 - 3e^{0}$$

$$= 1 - 3$$

$$= -2$$

$$P(0) = 50$$

$$y - 50 = -2(x)$$

$$y = -2x + 50$$

$$y \le 40$$

$$-2x + 50 \le 40$$

$$-2x \le -10$$

$$x \ge 5$$

It predicts that at $t = 5$ the lake will become safe.

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C1

Work for problem 2(a)

$P'(9) = 1 - 3e^{-0.2\sqrt{9}} = -0.646$ gallons/day

the level of pollutant is decreasing because the rate is negative, as it is decreasing.

Work for problem 2(b)

gallons of pollutant at a min when $P'(t) = 0$

$1 - 3e^{-0.2\sqrt{t}} = 0$

$3e^{-0.2\sqrt{t}} = 1$

$e^{-0.2\sqrt{t}} = 1/3$

$-0.2\sqrt{t} = \ln 1/3$

$\sqrt{t} = \frac{\ln 1/3}{-0.2}$

$t = \left(\frac{\ln 1/3}{-0.2}\right)^2$

$= 30.174$

≈ 30 days

C_2

Work for problem 2(c)

no of gallons present at the lake

$= 50 + \int_0^{30.174} (1 - 3e^{-0.2\sqrt{t}})\,dt$

$= 50.000$ gallons.

the lake is not safe because the no. of gallons is above 40 gallons.

Work for problem 2(d)

slope of tangent $= 1 - 3e^{-0.2\sqrt{t}}$

at $t=0$; gallons $= 50$ $\Rightarrow$ $m_T\big|_{(0,50)} = 1 - 3e^{-0.2\sqrt{0}} = -2$

equation of tangent: $y = -2x + 50$

40 lake is safe $\Rightarrow$ $40 = -2x + 50$

$\Rightarrow -2x = -10$

$\Rightarrow x = 5$

after 5 days

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NO CALCULATOR ALLOWED

A,

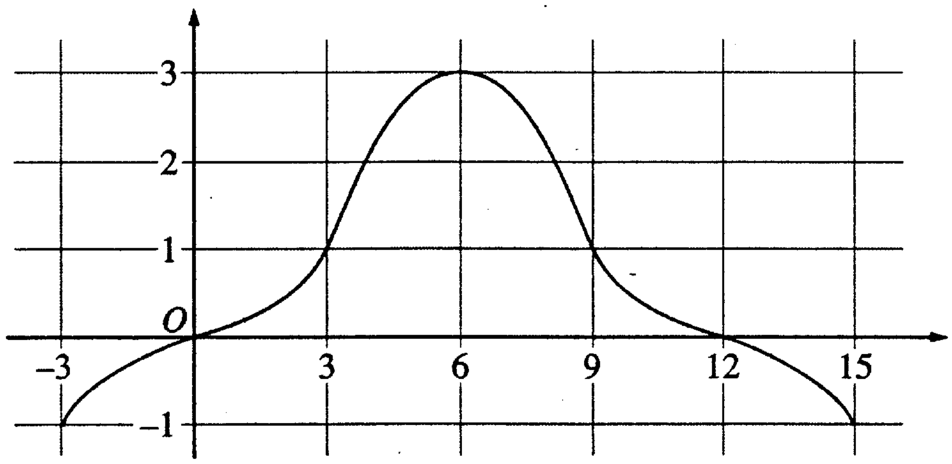
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

Work for problem 4(a)

$g(6) = 5 + \int_6^6 f(t)\,dt = 5$

$g'(6) = f(6) = 3$

$g''(6) = f'(6) = 0$

Work for problem 4(b)

$g'(x) = \frac{d}{dx}\int_6^x f(t)\,dt = f(x)$

g decreases when $f(x) < 0$.

$-3 < x < 0, \quad 12 < x < 15$

NO CALCULATOR ALLOWED

A2

Work for problem 4(c)

$g''(x) = f'(x) < 0$

$f'(x) < 0$ when $f(x)$ is decreasing

$6 < x < 15$

Work for problem 4(d)

$3 \times \left(\frac{-1+0}{2}\right) + 3 \times \left(\frac{0+1}{2}\right) + 3 \times \left(\frac{1+3}{2}\right) + 3 \times \left(\frac{3+1}{2}\right) + 3 \times \left(\frac{1+0}{2}\right) + 3 \times \left(\frac{0+(-1)}{2}\right)$

$= 3 \times 4 = 12$

GO ON TO THE NEXT PAGE.

 C,

NO CALCULATOR ALLOWED

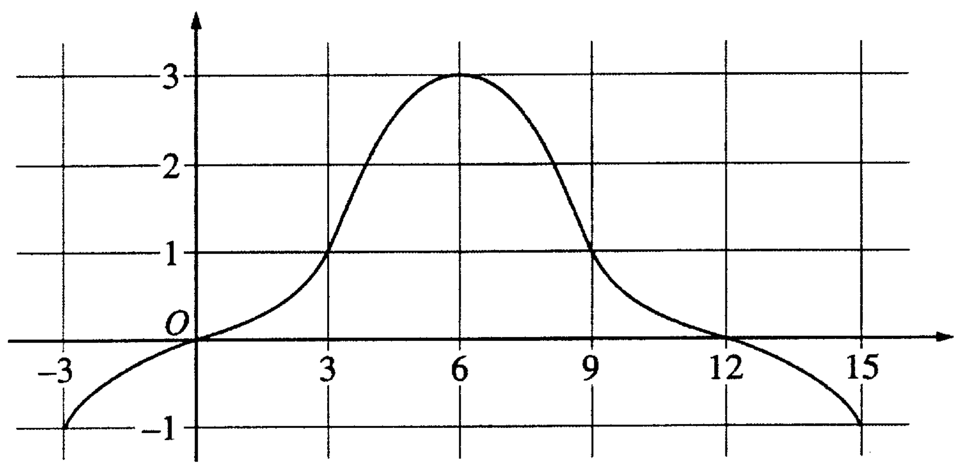
CALCULUS

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

Work for problem 4(a)

$g(6) = 5 + \int_6^6 f(t)\,dt = 0$

$g'(x) = f(x)$

$\therefore\ g'(6) = f(6) = 3$

$g''(6) = f'(6) = 0$

Work for problem 4(b)

$g'(x) = f(x)$ from $g'(x) = 0 + \frac{dg}{dx}\left[\int_6^x f(t)\,dt\right]$

$f(x) < 0$ on $-3 < t < 0$ and $12 < t < 15$

$\therefore\ g(x)$ is decreasing on $-3 < t < 0$ and $12 < t < 15$

C2

Work for problem 4(c)

$g''(x) = f'(x)$

$f'(x) < 0$ on $6 < t < 15$

$\therefore g(x)$ is concave down on $6 < t < 15$

Work for problem 4(d)

$A \approx \frac{18}{12}\left(|-1| + (1)(2) + (5)(2) + (1)(2) + |-1|\right)$

$\approx \frac{18}{12}(12) \approx 18$ squared units

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A1

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NO CALCULATOR ALLOWED

Work for problem 5(a)

1) Solving the equation

$\frac{dy}{dx} = \frac{3-x}{y}$

$y\,dy = (3-x)\,dx$

$\int y\,dy = \int (3-x)\,dx$

$\frac{y^2}{2} = 3x - \frac{x^2}{2} + C \quad y^2 = 6x - x^2 + C$. General solution.

2) Because $y=-2$ is tangent to $f(x)$ at $(x_0; -2)$,

$\left.\frac{dy}{dx}\right|_{\substack{x=x_0\\ y=-2}} = 0. \quad \frac{3-x_0}{-2} = 0$

$x_0 = 3.$

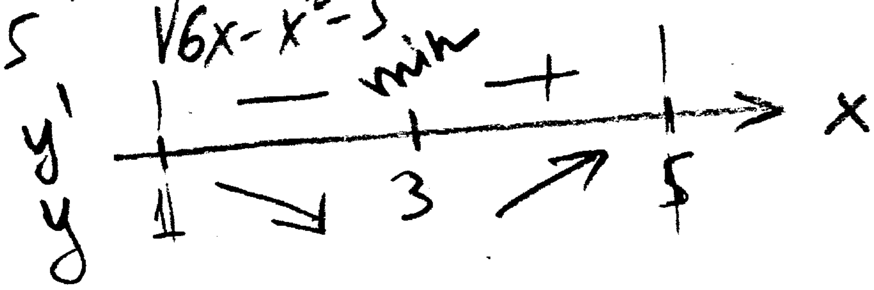
3) Particular solution $y = -\sqrt{6x}$.

$(3;-2)\quad 4 = 6\cdot 3 - 9 + C$

$C = -5$

4) $y' = \frac{-(3-x)}{\sqrt{6x-x^2-5}} = \frac{x-3}{\sqrt{6x-x^2-5}}$

at $x=3$



at $x=3$ $y=f(x)$ has a local minimum.

A2

NO CALCULATOR ALLOWED

Work for problem 5(b)

$\frac{dy}{dx} = \frac{3-x}{y}$

$y^2 = 6x - x^2 + C$ as found in a)1).

$y(6) = -4 \quad 16 = 6\cdot(+6) - 36 + C$

$\therefore C = 16$

$y^2 = 6x - x^2 + 16$

$y = -\sqrt{6x - x^2 + 16}$

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D1

5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED

Work for problem 5(a)

$1 < x < 5$

$y' = \frac{3-x}{y}$

$\frac{dy}{dx} = \frac{3-x}{y}$

$0 = \frac{3-x}{y}$

$0 = 3 - x$

~~point of tangency~~ $x = 3$

$3 = x$ -coordinate of point of tangency, local maximum

$y = \pm\sqrt{6x - x^2}$

$x = 0$
$y = 0$

inc

$\frac{2}{\sqrt{5}}$

$x = 1$ $y = \sqrt{5}$

$x = 4$ $y = 8$ de

I	$f'(x)$	
$-\infty, 3$	inc	
$(3, \infty)$	dec	

$\frac{-1}{8}$ c

$y = \pm\sqrt{6-1}$

D2

NO CALCULATOR ALLOWED

Work for problem 5(b)

(6, -4)

~~$\int y\,dy = \int 3-x\,dx$~~

$\frac{1}{2}y^2 =$

$\int y\,dy = \int 3-x\,dx$

$\frac{1}{2}y^2 + c = 3x - \frac{1}{2}x^2 + c$

$y = \sqrt{6x - x^2 + c}$

$-4 = \sqrt{36-36+c}$

$-4 \pm \sqrt{c}$

$g(x) = -\sqrt{6x-x^2+16}$

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 A1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$\ln\left(\frac{1}{1-x}\right) \Rightarrow \sum_{n=1}^{\infty} \frac{x^n}{n}$

$\ln\left(\frac{1}{1+3x}\right) \Rightarrow \sum_{n=1}^{\infty} \frac{(-3x)^n}{n}$ $\quad \left|\frac{(-3x)^{n+1}}{n+1} \cdot \frac{n}{(-3x)^n}\right| = \left|\frac{(-3x)n}{n+1}\right| =$

$|x| < \frac{1}{3}$

$-\frac{1}{3} < x \leq \frac{1}{3}$

(when $x = \frac{1}{3}$ converge
(when $x = -\frac{1}{3}$ diverge

Work for problem 6(b)

$\ln\left(\frac{1}{1-x}\right) \Rightarrow \sum_{n=1}^{\infty} \frac{x^n}{n}$

when $x = -1$

$\ln\left(\frac{1}{2}\right) = -\ln 2$

A2

NO CALCULATOR ALLOWED

Work for problem 6(c)

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converge at $0<p\leq 1$

$\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ diverge at $0.<p\leq \frac{1}{2}$

$\therefore \; 0<p\leq \frac{1}{2}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converge by alternating series test $\boxed{p=\frac{1}{2}}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test

Work for problem 6(d)

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges $\propto p\leq 1$

$\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges $\frac{1}{2}<p$

$\therefore \frac{1}{2}<p\leq 1$

$\boxed{p=1}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test.

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series test

D1

NO CALCULATOR ALLOWED

Work for problem 6(a)

Since $\ln\left(\frac{1}{1-x}\right) = x + \frac{x^2}{2} + \frac{x^3}{3} \ldots \sum_{n=1}^{\infty} \frac{x^n}{n}$,

$\ln\left(\frac{1}{1+3x}\right) = \ln\left(\frac{1}{1-(-3x)}\right) = -3x + \frac{(-3x)^2}{2} + \frac{(-3x)^3}{3} \ldots \sum_{n=1}^{\infty} \frac{(-3)^n x^n}{n}$

Interval of convergence:

$|-3x| < 1$

$x > -\frac{1}{3}$

$x < \frac{1}{3}$

$\boxed{-\frac{1}{3} < x \leq \frac{1}{3}}$

Work for problem 6(b)

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \Rightarrow$ alternating harmonic series $\Rightarrow$ converges

$= -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} \ldots = \boxed{0}$

NO CALCULATOR ALLOWED

Work for problem 6(c)

$p = \frac{1}{3}$,

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$ will converge b/c of its similarity to the alternating harmonic series.

$\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ is a divergent p-series since $p < 1$.

Work for problem 6(d)

$p = 1$.

$\sum_{n=1}^{\infty} \frac{1}{n^1} \Rightarrow$ divergent harmonic series.

$\sum_{n=1}^{\infty} \frac{1}{n^2} \Rightarrow$ convergent p-series since $p > 1$.

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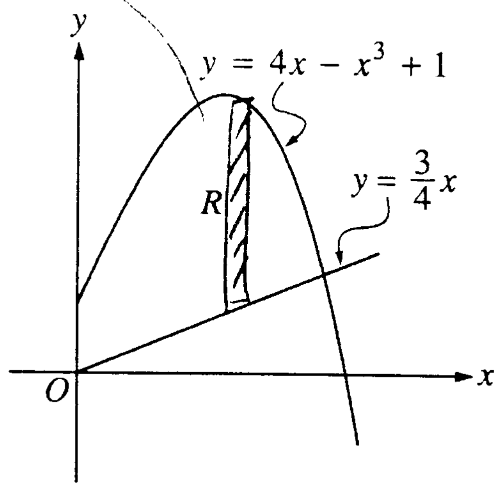
3 3 3 3 3 3 3 3 3 3 B1

elemental strip
$(4x - x^3 + 1) - (\frac{3}{4}x)$

dx

$dv = (4x - x^3 + 1) - (\frac{3}{4}x)\, dx$



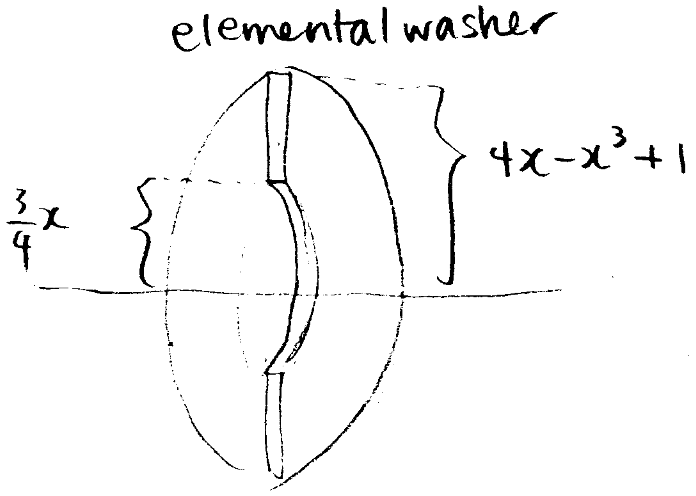
Work for problem 3(a)

intersection: (1.9404, 1.4553)

$$A_R = \int_0^{1.9404} (4x - x^3 + 1) - (\frac{3}{4}x)\, dx$$

$$= \boxed{4.515 \text{ units}^2} \text{ (calculater)}$$

Work for problem 3(b)

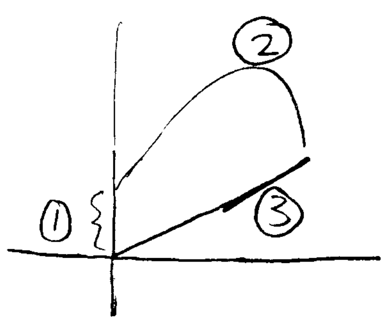


$$V_R = \pi \int_0^{1.9404} (4x - x^3 + 1)^2 - \left(\tfrac{3}{4}x\right)^2 \, dx$$

$$= \boxed{57.463 \text{ units}^3}$$

$$dv = \pi\left((4x - x^3 + 1)^2 - \left(\tfrac{3}{4}x\right)^2\right) dx$$

Work for problem 3(c)



$$① = 1$$

$$② = \int_0^{1.9404} \sqrt{1 + [f'(x)]^2} \, dx$$

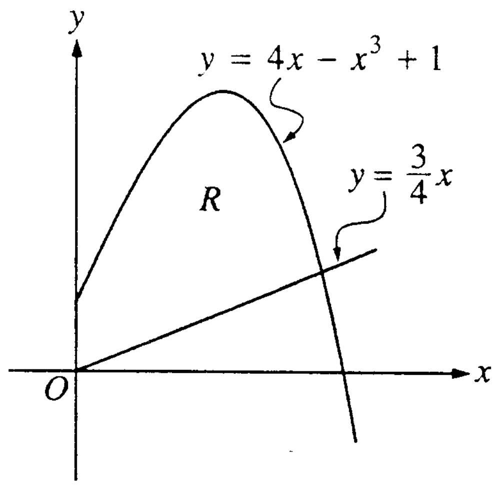
$$= \int_0^{1.9404} \sqrt{1 + [4 - 3x^2]^2} \, dx$$

$$③ = \sqrt{(1.4553 - 0)^2 + (1.9404 - 0)^2}$$

$$= 2.4255$$

$$\boxed{\text{perimeter} = \int_0^{1.9404} \sqrt{1 + [4 - 3x^2]^2} \, dx + 3.426}$$

D1



Work for problem 3(a)

$$= \int_0^{1.9404496} \left[(4x - x^3 + 1) - \left(\tfrac{3}{4}x\right)\right] dx$$

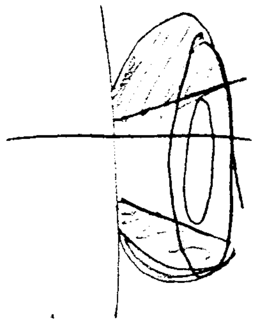
$$= \int_0^{1.9404496} 4x - x^3 + 1 \; dx \; - \int_0^{1.9404496} \frac{3}{4}x \; dx$$

$$= 2x^2 - \tfrac{1}{4}x^4 + x \Big|_0^{1.9404496} - \frac{3}{8}x^2 \Big|_0^{1.9404496}$$

$$= 5.9266838 - 1.4120042$$

$$= 4.515 \text{ units}^2$$

Work for problem 3(b)



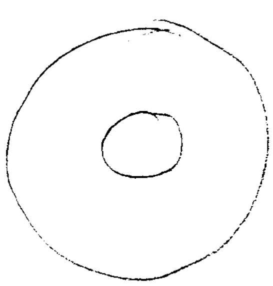
$V = \pi r^2$

$V = \pi \int_0^{1.9404496} (4x - x^3 + 1)^2 - \left(\frac{3}{4}x\right)^2 \, dx$

$= 19.661088\pi - 1.3699615\pi$

$= 18.291\pi$ units^3.

Work for problem 3(c)



like a bowl.

$= 2\pi \int_0^{1.9404496} \sqrt{1 + (4 - 3x^2)^2} \, dx + 2\pi \int_0^{1.9404496} \sqrt{1 + \left(\frac{3}{4}x\right)^2} \, dx$